Illustrative Mathematics

3.NF Which is Closer to 1?

Alignment 1: 3.NF.A.2

Which is closer to 1 on the number line, $\frac{4}{5}$ or $\frac{5}{4}$? Explain.

Commentary:
This can be seen as a multi-step problem for grade 3:

- compare $\frac{4}{5}$ to $\frac{5}{5}$ (like denominators)
- compare $\frac{4}{4}$ to $\frac{5}{4}$ (like denominators)
- compare $\frac{1}{4}$ to $\frac{1}{5}$ (like numerators)

This task is also a natural fit for 4.NF.2 as a lower-level task in a set that would illustrate that standard. Thus, this task provides a nice transition between grade levels.
Although a number line diagram is not provided, many students may choose to draw one; teachers might suggest doing so if a student is struggling with the problem. For many students, creating a number line will help them recognize that $\frac{4}{5}$ is only $\frac{1}{5}$ from 1 and that $\frac{5}{4}$ is only $\frac{1}{4}$ from 1 even though the relative size of the fractions is similar and the two fractions are plotted on different sides of 1 on the number line. Some students may choose to create two number lines, so they can partition one into fourths and one into fifths, and then compare them.

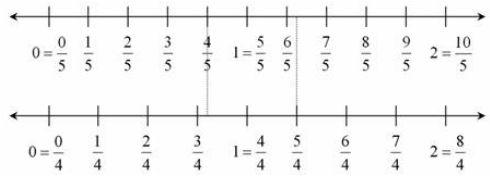
It is possible to solve this problem using common denominators. While not technically incorrect, that solution is not shown because first, students do not work with common denominators until fourth grade, and second, even when they know how to find common denominators, they should recognize in a case like this that straight-forward reasoning about the relative sizes of unit fractions is more efficient.

Below is a list of related tasks in this set in order of sophistication:

- Locating Fractions Less than One on the Number Line
- Locating Fractions Greater than One on the Number Line
- Closest to $\frac{1}{2}$
- Find 1
- Find $\frac{2}{3}$
- Which is Closer to 1?

Solution: 3.NF.2 Plot Points to Compare

Many students will sketch something like the following number line diagram:



Fifths are closer together than fourths. $\frac{4}{5}$ is $\frac{1}{5}$ from 1. $\frac{5}{4}$ is $\frac{1}{4}$ from 1. So $\frac{4}{5}$ is closer to 1 than $\frac{5}{4}$.

Solution: 3.NF.2 Recognize and Reason

A few students may simply reason in the following way (stated in student language rather than the more formal language used here):

If we divide the segment from 0 to 1 in four equal-length segments (so each has a length of $\frac{1}{4}$), and we also divide the segment from 0 to 1 into five equal-length segments (so each has a length of $\frac{1}{5}$), the segments of length $\frac{1}{4}$ will be longer than the segments of length $\frac{1}{5}$ because there are fewer of them.

The distance between $\frac{4}{5}$ and 1 is $\frac{1}{5}$, and the distance between $\frac{5}{4}$ and 1 is $\frac{1}{4}$.

So $\frac{4}{5}$ is closer to 1 than $\frac{5}{4}$

A correct solution does require that the student explain how he or she figured out that $\frac{4}{5}$ is closer to 1.